5. Wind intermittency parameters

There are three basic intermittency parameters for wind that are calculated for each period in WinDS before the linear program optimization is conducted for that period. These include wind capacity value (WCV), wind operating reserve (WOR), and wind surplus (WSurplus). For each, a marginal value is calculated, which applies to new wind installed in the period, and an "old" value is calculated, which applies to all the wind built in previous periods. This section describes the statistical assumptions and methods used to calculate these values.

Wind Capacity Value

This is the capacity credit given to the wind contribution to meeting the reserve margin constraint in each NERC region. It is a function of the amount of wind consumed in the NERC region, the dispersion of the wind farms contributing that wind, the electric load in the NERC region, and the amount and reliability of conventional capacity contributing to the load in the NERC region. Generally, as more wind is used by the NERC region, the wind capacity value decreases.

For each additional MW of class c wind added in region i that is consumed in NERC region r, the capacity credit, $WCVmar_{c,i,r}$, is the amount of load that can be added in every hour without changing the loss of load probability in NERC region r. To calculate $WCVmar_{c,i,r,}$ we assume that for each hour in the period for which WCVmar is being evaluated, the sum of all the conventional generation consumed in the NERC region r, plus all the wind generation from all classes of existing wind consumed in the NERC region r, minus the load in the NERC region r is a random variable that can be well approximated by a normal probability distribution. This approximation improves in accord with the central limit theorem as the number of contributing conventional plants and wind farms increases. Let X be this random variable. Then

$$\mu_x = \mu_C + \mu_W - \mu_L$$

Where:

μ denotes expected value
C is generation from all conventional power plants that deliver power to NERC region r
W is generation from all wind farms that deliver power to NERC region r
L is the load in NERC region r

And since C, W, and L are statistically independent

$$\sigma^2_X = \sigma^2_C + \sigma^2_W + \sigma^2_L$$
$$\sigma_X = (\sigma^2_C + \sigma^2_W + \sigma^2_L)^{0.5}$$

Where σ denotes standard deviation and σ^2 is the variance

The loss of load probability is the probability that X is less than zero or P(X<0). Define $X' = (X - \mu_x)/ \sigma_X$ as a standard normal variable. Then P(X<0) is the probability that X' is less than $-\mu_x / \sigma_X$ or $N(-\mu_x / \sigma_X)$ where N is the cumulative standard normal distribution function.

To calculate how much load LD can be taken on in NERC region r with the next increment $WD_{c,i}$ of class c wind supplied from wind region i without changing the loss of load probability, we now define the normal random variable $U = C + (W + WD_{c,i}) – (L + LD)$ with

$$\mu_U = \mu_C + \mu_W + \mu_{WDc,i} - \mu_L - \mu_{LD}$$

And $$\sigma^2_U = \sigma^2_C + \sigma^2_{W+WDc,i} + \sigma^2_{L+LD}$$

Where:

$WD_{c,i}$ is the generation from a small positive class c wind capacity addition in region i
LD is the increase in NERC region r load that maintains the loss of load probability.
Then the effective load carrying capacity of wind is $LD/WD_{c,i}$ when $P(U<0) = P(X<0)$
where $WD_{c,i} = \mu_{WDc,i} /CF$ with CF being the capacity factor of the class c wind capacity addition.

Finally, define the standard normal random variable $U' = (U - \mu_U)/ \sigma_u$. Then $P(U < 0) = P(U' < - \mu_U / \sigma_u = N(-\mu_U / \sigma_u)$. Then the effective load-carrying capacity of additional class c wind from region i supplied to NERC region r is $WCVmar_{c,I,r} = LD/ WD_{c,i}$ when

$$P(U<0) = P(X<0)$$

Or when

$$N(-\mu_U / \sigma_u) = N(-\mu_x / \sigma_X)$$

Or

$$- \mu_U / \sigma_u = - \mu_x / \sigma_X$$

Or

$$\mu_U / \sigma_u = \mu_x / \sigma_X$$

Or

$$[\mu_C + \mu_W + \mu_{WDc,i} - \mu_L - \mu_{LD}] / \sigma_u = \mu_x / \sigma_X$$

Or

$$\mu_{LD} = \mu_C + \mu_W + \mu_{WDc,i} - \mu_L - \mu_x * \sigma_u / \sigma_X$$

Or since the load added is not a random variable

$$LD = WD_{c,i} *CF + \mu_x * [1- \sigma_u / \sigma_X]$$

Thus, $WCVmar_{c,i,r} = LD / WD_{c,i} = CF - [\sigma_u / \sigma_X - 1] * \mu_x / WD_{c,i}$

Because the capacity value of wind is most important during the peak load periods, $WCVmar_{c,i,r}$ is calculated using the capacity value of wind during the peak load period of the NERC region. Similarly, the generation from conventional plants is assumed to be that of the peak load period, i.e. accounting for forced outages, but not planned outages, because they are generally planned for off-peak periods.

$$\mu_C = \sum_q CONVCAP_{q,n} * (1 - fo_q)$$

And $$\sigma^2{}_C = \sum_q numplants_{q,r} * plantsize_q^{\,2} * fo_q * (1 - fo_q)$$

Where $numplants_{q,r} * plantsize_q = CONVCAP_{q,n}$

The expected load μ_L is assumed to be the input load for the NERC region.

The variance of the load σ^2_L is derived from the load-duration curve associated with a NERC region.

The variance of the wind generated by all the wind farms contributing to NERC region r is built up from the variance of the output of each individual wind farm and the covariance between those outputs. The standard deviation of the output of an individual wind farm is assumed to be the sum of the standard deviations of the outputs of the individual turbines in the wind farm, i.e. the outputs of the turbines within a wind farm are assumed to be perfectly correlated. The distribution of the wind itself is approximated with a Weibull distribution with shape parameter k = 2. The second parameter of the Weibull distribution of the wind speed is adjusted to ensure that the annual output of the wind turbine will produce the annual capacity factor of the wind turbine in the year 2000. The wind speed is translated to power output using the power curve of an individual wind turbine (Vestas 1650). Thus, the distribution on wind speed is translated to a distribution on wind power output. The variance in the wind power output is calculated from this wind power distribution.

WinDS assumes that the capacity factor associated with each class of wind power resource will improve with technological innovation in years after 2000. To translate the improved capacity factors into new estimates of the variance in the wind power distribution, a regression is used with the 2000 capacity factors as the independent variables that determine the variance, the dependent variables.

The variance of the wind output of individual wind farms is used to estimate the variance σ^2_{Wr} of the output from all wind farms contributing to the electric loads of a NERC region.

$$\sigma^2_{W_r} = \sum_{c,i} \sigma^2_{W_{c,i,r}} + 2\sum_{\substack{c,i \\ cc,ii}} COVAR_{c,i,cc,ii.r}$$

Where $COVAR_{c,i,cc,ii,r}$ is the covariance between class c wind in region i and class cc wind in region ii.

$$COVAR_{c,i,cc,ii,r} = Corr_{c,i,cc,ii,r} * \sigma_{W_{c,i}} * \sigma_{W_{cc,ii}}$$

Where:

$CORR_{c,i,cc,ii,r}$ is the correlation between class c wind in region i and class cc wind in region ii. This correlation is assumed to be a linear function of the distance between the center of region i and that of region ii with full correlation (+1) of all wind of the same class in the same region and zero correlation at 500 miles separation. The correlation between two different wind classes in the same region is only 75%, under the assumption that two wind farms with different classes, although in the same region, are not collocated.

$\mathbf{WCVold_r}$: The capacity value of wind built in previous periods, $WCVold_r$, is calculated each period. $WCVold_r$ is the amount of load that must be dropped per MW of wind to retain the same loss of load probability, if the wind is no longer available in NERC region r.

As with $WCVmar_{c,i,r}$, we define the random variable X as the sum of the conventional and wind generation minus the load.

$\mu_x = \mu_C + \mu_W - \mu_L$

Where:

μ denotes expected value
C is generation from all conventional power plants that deliver power to NERC region r
W is generation from all wind farms that deliver power to NERC region r
L is the load in NERC region r

If CF_r is defined as the average capacity factor of all the wind farms that deliver wind to NERC region r, then $\mu_W = WD_r * CF_r$

Since C, W, and L are statistically independent

$$\sigma^2_X = \sigma^2_C + \sigma^2_W + \sigma^2_L$$
$$\sigma_X = (\sigma^2_C + \sigma^2_W + \sigma^2_L)^{0.5}$$

Where σ denotes standard deviation and σ^2 is the variance

The loss of load probability is the probability that X is less than zero or P(X<0). Define $X' = (X - \mu_x)/\sigma_X$ as a standard normal variable. Then the probability that X is less than zero is the probability that X' is less than $-\mu_x / \sigma_X$ or $N(-\mu_x / \sigma_X)$ where N is the cumulative standard normal distribution function.

To calculate $WCVold_r$, we define the normal random variable U = C– (L - LD) similar to X in the calculation of $WCVmar_{c,i,r}$ except without the wind contribution. Then

$$\mu_U = \mu_C - \mu_{L-LD}$$

And $$\sigma^2_U = \sigma^2_C + \sigma^2_{L-LD}$$

Then the effective load-carrying capacity of all the wind supplied to NERC region r is $WCVold_r = LD/WDC_r$ when

$$P(U<0) = P(X<0)$$

Or when

$$N(-\mu_U / \sigma_u) = N(-\mu_x / \sigma_X)$$

Or

$$- \mu_U / \sigma_u = - \mu_x / \sigma_X$$

Or

$$\mu_U / \sigma_u = \mu_x / \sigma_X$$

Or

$$[\mu_C - \mu_L + \mu_{LD}] / \sigma_u = \mu_x / \sigma_X$$

Or

$$\mu_{LD} = \mu_L - \mu_C + \mu_x * \sigma_u / \sigma_X$$

Or since the load subtracted (LD) is not a random variable

$$LD = \mu_L - \mu_C + \mu_x * \sigma_u / \sigma_X$$

Thus

$$WCVold_r = LD/WDC_r = (\mu_L - \mu_C + \mu_x * \sigma_u / \sigma_X) / WDC_r$$

Or

$$WCVold_r = (\mu_L - \mu_C - \mu_W + \mu_W + \mu_x * \sigma_u / \sigma_X) / WDC_r$$

Or

$$WCVold_r = (\mu_W - \mu_x *(1 - \sigma_u / \sigma_X)) / WDC_r$$

Or

$$WCVold_r = CF_r - \mu_x *(1 - \sigma_u / \sigma_X)) / WDC_r$$

Or

$$WCVold_r = CF - \mu_x *(1 - \sigma_u / \sigma_X)/WDC_r$$

which looks very similar to the calculation of $WCVmar_{c,i,r}$

$WORmar_{c,i,r}$

$WORmar_{c,i,r}$ is the operating reserve requirement induced by the next MW of class c wind installed in region i that contributes generation to NERC region r. It is calculated as the difference in the operating reserve required with an increment $WD_{c,i,r}$ of additional wind capacity, minus that required with only the existing wind with the difference divided by the incremental wind capacity $WD_{c,i,r}$.

$$WORmar_{c,i,r} = (\sqrt{NOR2 + wor2factor * \sigma^2_{W_r + WD_{c,i,r}}} - \sqrt{NOR2 + wor2factor * \sigma^2_{Wr}}) / WD_{c,i,r}$$

Where:

NOR2 is the variance of the usual operating reserve requirement

$$NOR2 = (Norfrac / resconf\text{int} * (\sum_{n \in r} P_n - Norhydro * \sum_{n \in r} Conv_{n,hydro}))^2$$

Norfrac is the normal operating reserve fraction per MW of load.

resconfint is the multiplier on the variance of the load required to yield an adequate confidence interval

P_n is the peak load in PCA n in NERC region r

Norhydro is the amount by which the operating reserve can be reduced for each MW of hydroelectricity in the region

wor2factor is a multiplier on the wind variance to provide the appropriate impact on operating reserve requirements

$\sigma_{Wr+WDc,i,r}$ is the standard deviation of the output from all the wind generation consumed in NERC region r and that from the incremental capacity $WD_{c,i,r}$

σ_{Wr} is the standard deviation of the output from all the wind generation consumed in NERC region r

$WORold_{c,r}$

$WORold_{c,r}$ is the average operating reserve induced per MW of existing class c wind that is consumed in NERC region r. It is calculated as the difference in the operating reserve required with the existing wind capacity, minus that required were no wind used, divided by the total wind capacity contributing to NERC region r, W_r.

$$\text{WORold}_{c,r} = (\sqrt{NOR2 + wor2factor * \sigma^2_{W_r}} - \sqrt{NOR2}) / W_r$$

IWSurplusOld$_{in}$

IWsurplusold$_{in}$ is the expected fraction of generation from all the wind consumed in interconnect "in" that cannot be productively used, because the load is not large enough to absorb both it and the must-run generation from existing conventional sources. This situation occurs most frequently in the middle of the night when loads are small, base-load conventional plants are running at their minimum levels, and the wind is blowing.

To calculate IWsurplusold$_{in}$, we define a new random variable Y as the sum of the random variables for wind generation W and the must-run conventional base-load generation M minus the load L.

Y = W + M – L

Next, we define the surplus wind at any point in time, SU, as

If Y < 0, SU = 0

If Y > 0, SU = Y

Then, the expected total surplus μ_{SU} can be calculated from its density function f(s) and the density function of y, g(y) as follows:

$$\mu_{SU} = \int_{-\infty}^{\infty} sf(s)ds$$

$$\mu_{SU} = \int_{-\infty}^{0} sf(s)ds + \int_{0}^{\infty} sf(s)ds$$

$$\mu_{SU} = \int_{-\infty}^{0} yfy)dy + \int_{0}^{\infty} yg(y)dy$$

$$\mu_{SU} = 0 + \int_{0}^{\infty} yg(y)dy$$

Now if we assume, as we did in the WCVmar and WORmar calculations above, that by the central limit theorem, Y can be well approximated by a normal distribution, and we define the standard normal variable Y' as

Y' = (Y – μ_Y)/ σ_Y

Then

Y = Y' * σ_Y + μ_Y, and

dy = σ_Y dY'

Thus

$$\mu_{SU} = \int_{-\mu_Y/\sigma_Y}^{\infty} (y'\sigma_Y + \mu_Y) * g(y'\sigma_Y + \mu_Y) * \sigma_Y dy'$$

$$\mu_{SU} = \int_{-\mu_Y/\sigma_Y}^{\infty} ((\sigma_Y)^2 * y' * g(y'\sigma_Y + \mu_Y))dy' + \int_{-\mu_Y/\sigma_Y}^{\infty} \mu_Y * \sigma_Y * g(y'\sigma_Y + \mu_Y)dy'$$

Assuming Y is normally distributed as stated above:

$$\mu_{SU} = \int_{-\mu_Y/\sigma_Y}^{\infty} ((\sigma_Y)^2 * y' * (1/\sigma_Y\sqrt{2\Pi}) * \exp(-(y'\sigma_Y + \mu_Y)^2 / (2\sigma_Y{}^2))dy'$$

$$+ \int_{-\mu_Y/\sigma_Y}^{\infty} \mu_Y * \sigma_Y (1/\sigma_Y\sqrt{2\Pi}) * \exp(-(y'\sigma_Y + \mu_Y)^2 / (2\sigma_Y{}^2))dy'$$

$$\mu_{SU} = \int_{-\mu_Y/\sigma_Y}^{\infty} (\sigma_Y * y' / \sqrt{2\Pi} * \exp(-y'^2/2)dy' + \int_{-\mu_Y/\sigma_Y}^{\infty} \mu_Y / \sqrt{2\Pi} * \exp(-y'^2/2)dy'$$
$$\mu_{SU} = \sigma_Y / \sqrt{2\Pi} * \exp(-\mu_Y^{\,2} / 2\sigma_Y^{\,2}) + \mu_Y(1 - N_{0,1}(-\mu_Y/\sigma_Y))$$

Where $N_{0,1}$ is the standard normal distribution with mean 0 and standard deviation 1.

Then $IWsurplusold_{in}$ is the difference between the expected surplus with wind, μ_{SU}, and the expected surplus were there no wind generation consumed in interconnect "in", μ_{SUN} , divided by the total wind capacity contributing to interconnect in, W_{in}. Or

$IWsurplusold_{in} = (\mu_{SU} - \mu_{SUN})/W_{in}$

Normally μ_{SUN} would be zero, as the conventional must run units would not be constructed in excess of the minimum load. However, with our assumption of a normal distribution for Y, we do introduce some non-zero probability that Y could be positive even if there were no wind, i.e. that the generation from must-run units could exceed load. Thus, it is important to calculate μ_{SUN} and to subtract it from μ_{SU} to remove the bulk of the error introduced by the normal distribution assumption. μ_{SUN} is calculated in exactly the same way as μ_{SU}, but with no wind included.

Must-run conventional capacity is defined as existing available (i.e., not in a forced outage state) coal and nuclear capacity times a minimum turn-down fraction, **MTDF**. The expected value of the must-run capacity of type q available at any given point in time, μ_{Mq} , is thus:

$\mu_{Mq} = CONVCAP_{q,in} * (1-FO_q) * MTDF_q$

Where:

> **$CONVCAP_{in,q}$** is the existing conventional capacity in interconnect in of type q
> **MTDF** is
> > 0.45 for old (built before the year 2000) coal plants
> > 0.35 for new coal plants (built in 2000 or later, i.e. built within the model run time frame)
> > 1.0 for nuclear plants

$IWSurplusMar_{c,i,in}$

$IWsurplusmar_{c,i,in}$ is the fraction of generation from the next MW of class c wind installed in wind supply region i destined for interconnect in that cannot be productively used because the load is not large enough to absorb both it and the must-run generation from existing conventional sources. It is calculated as:

$IWsurplusmar_{c,i,in} = (\mu_{SUi} - \mu_{SU})/100$

Where μ_{SUi} is calculated in exactly the same way as μ_{SU}, but with 100 MW of wind added in region i.